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TURBULENT EXCHANGE OF MOMENTUM, MASS, AND HEAT BETWEEN FLUID STREAMS AND PIPE WALL

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Abstract—A new correlation is presented to describe mass and heat transfer to a fluid in a fully developed turbulent flow in a pipe. The correlation differs from earlier empirical relations in that it is based on a theoretical continuous eddy-viscosity distribution from the wall to the center of the pipe. Transfer rates calculated from the new correlation are in excellent agreement with experimental data on mass and heat transfer to fluid streams.

NOMENCLATURE

- C, concentration of diffusing species;
- C_W, concentration at the wall;
- C_{avg}, time-average concentration;
- C_p, heat capacity;
- D, molecular diffusivity;
- F, functional notation;
- g_c, conversion constant;
- k_c, mass-transfer coefficient;
- Nu, Nusselt number;
- Pr, Prandtl number;
- Sc, Schmidt number;
- St, Stanton number;
- T_W, temperature at the wall;
- T, temperature;
- T_{avg}, average temperature;
- U^+, dimensionless velocity;
- y, distance from the wall;
- y^+, yU_τ/ν;
- μ, molecular viscosity;
- ν, kinematic viscosity;
- ρ, density of fluid;
- ϵ_v, eddy viscosity;
- ϵ_d, eddy diffusivity for mass;
- ϵ_c, eddy conductivity;
- τ_W, shear stress at the wall;
- St_m, Stanton number for mass;
- St_h, Stanton number for heat;
- u, fluctuating velocity in the axial direction;
- v, fluctuating velocity in the radial direction;
- $\overline{uv}$, Reynolds stress.

INTRODUCTION

THE turbulent exchange of momentum, mass, and heat in the vicinity of a boundary is encountered in many engineering processes. It is well established [1, 2, 3] that in the vicinity of a boundary the turbulent exchange of momentum, mass, and heat is governed not only by molecular motion but also by an eddy motion. To predict rates of mass and heat transfer between a moving fluid and a wall it is therefore essential to understand the mechanism of eddy motion in the vicinity of a wall. Unfortunately very little is known about the distribution of eddies very close to the wall. In the past, due to the absence of both theoretical analyses and experimental data for the region close to the wall, a number of empirical expressions have been proposed for eddy-viscosity distributions near a wall. An excellent review of several such existing correlations for fully developed turbulent flow in a pipe with constant fluid properties has been presented by Sherwood [3].

Recently Wasan, Tien, and Wilke [4] pointed out that most of the proposed eddy-viscosity distributions do not satisfy the theoretical criterion which states that the turbulent contribution to Reynolds stress $\overline{uv}$ near the wall is proportional to y^n where n is not less than three. This criterion was first derived by Townsend [5]. Also, all the previous analyses are based on the concept of three sharply defined fluid layers, namely laminar sublayer, buffer, and turbulent layers. However, according to these authors [4], and also Gowariker [6], this concept of three different fluid layers leads to an unrealistic discontinuity in the value of the eddy-viscosity function with respect to that obtained from logarithmic distribution in the turbulent core. Rannie [7] and Sleicher [8] in their analyses avoided this point of discontinuity, but the eddy-viscosity functions of both of these authors do not give satisfactory relationships for analogy expressions for heat- and mass-transfer rates for systems with high Schmidt or Prandtl numbers. From velocity-variation data and from turbulent shear-stress data of Laufer [9] it is evident that the degree of turbulence in the moving fluid varies continuously from the wall to the axis of a pipe. Hence, the concept of three distinct fluid layers would appear to be incorrect.

By using the equations of mean motion and the well established empirical logarithmic velocity distribution in the turbulent core, Wasan, Tien, and Wilke [4] have derived theoretical expressions for the continuous variation of velocity and eddy viscosity for the wall region of pipe flow. The distributions of these authors fit the experimental data on velocity and turbulent shear stress over the wall region. Their velocity and eddy-viscosity distributions for the wall region ($0 \leqslant y^+ \leqslant 20$) are presented in equations (1) and (2) as follows:

$$U^+ = y^+ - 1{\cdot}04 \times 10^{-4} y^{+4} + 3{\cdot}03 \times 10^{-6} y^{+5}, \tag{1}$$

and

$$\frac{\epsilon_v}{\nu} = \frac{4{\cdot}16 \times 10^{-4} y^{+3} - 15{\cdot}15 \times 10^{-6} y^{+4}}{1 - 4{\cdot}16 \times 10^{-4} y^{+3} + 15{\cdot}15 \times 10^{-6} y^{+4}} \tag{2}$$

We present correlations relating the fluid friction and turbulent-exchange rates of mass and heat over a wide range of Schmidt and Prandtl numbers based on these distributions. The corresponding concentration and temperature distributions for the wall region of pipe flow are also presented.

ANALYSIS

Consider a fully developed turbulent flow of fluid with constant properties in a pipe having walls kept at a constant concentration C_W or at a constant temperature T_W. Mass or heat is transferred to the fluid stream both by molecular and by eddy motions. As is customary, the shear–stress, mass-transfer, and heat-transfer fluxes can be written as the sum of molecular and turbulent fluxes as follows:

$$\tau g_c = (\mu + \rho \epsilon_v) \frac{dU}{dy}, \tag{3}$$

$$N_A = (D + \epsilon_d) \frac{dC}{dy}, \tag{4}$$

and

$$q = \rho cp(k + \epsilon_c) \frac{dT}{dy}, \tag{5}$$

where τ is the shear stress at a plane parallel to the wall, N_A is the mass flux, and q is the heat flux across a plane parallel to the wall.

To solve (3), (4) and (5), several assumptions are usually made [3]. First, in the wall region the shear stress τ, mass-transfer flux N_A, and heat-transfer flux q are assumed to be constant and equal to the value at the wall. Second, in the fully turbulent region the variation of shear stress, mass transfer, and heat transfer is such that τ/N_A and τ/q are constant, and molecular viscosity μ, molecular diffusivity D and thermal conductivity k can be neglected. Third, in the case of mass transfer the interfacial velocity is assumed to be negligible. The last important assumption is that eddy diffusivities of momentum ϵ_v, mass ϵ_d and heat ϵ_c are equal.

Now, by combinations of (3) and (4) and after integration, for the turbulent region one obtains

$$\frac{C_{avg} - C_1}{U_{avg} - U_1} = \frac{\rho N_{AW}}{\tau_W g_c}, \tag{6}$$

where C_{avg} and U_{avg} refer to the average concentration and velocity respectively, and C_1 and U_1 refer to concentration and velocity each corresponding to $y^+ = 20$. After combining (1) and (6) one gets

$$C_{avg} - C_1 = \frac{\rho}{\tau_W g_c} N_{AW}\, U_{avg}[1 - 13{\cdot}0\sqrt{(f/2)}], \tag{7}$$

where the concentration C_1 corresponding to $y^+ = 20$ is determined as follows:

In the wall region equation (4) can be rewritten as

$$N_{AW} = \sqrt{\left(\frac{\tau_W g_c}{\rho}\right)} \left(\frac{1}{Sc} + \frac{\epsilon_v}{\nu}\right) \frac{dC}{dy^+}. \quad (8)$$

Integration of (8) gives

$$C_1 - C_W = \frac{N_{AW}}{\sqrt{(\tau_W g_c/\rho)}} \int_0^{20} \frac{dy^+}{(1/Sc + \epsilon/\nu)}. \quad (9)$$

Therefore, from (7) and (9) there results

$$C_{avg} - C_W = \frac{\rho N_{AW}}{\tau_W g_c} U_{avg} [1 - 13{\cdot}0 \sqrt{(f/2)}] + \frac{N_{AW}}{\sqrt{(\tau_W g_c/\rho)}} \int_0^{20} \frac{dy^+}{(1/Sc + \epsilon/\nu)}. \quad (10)$$

But the mass flux N_{AW} at the wall can be given by the expression

$$N_{AW} = k_c (C_{avg} - C_W). \quad (11)$$

Hence, from (10) and (11) the expression for mass-transfer Stanton number in terms of Schmidt number and friction factor becomes

$$St_m = \frac{k_c}{U_{avg}} = \frac{f/2}{1 + \sqrt{(f/2)}\left[\int_0^{20} \frac{dy^+}{(1/Sc + \epsilon_v/\nu)} - 13{\cdot}0\right]}, \quad (12)$$

where ϵ/ν is given by equation (2).

Similarly from (3) and (5) the expression for heat-transfer Stanton number in terms of Prandtl number and friction factor can be obtained as

$$St_h = \frac{f/2}{1 + \sqrt{(f/2)}[F(Pr, 20) - 13{\cdot}0]}. \quad (13)$$

CONCENTRATION AND TEMPERATURE DISTRIBUTIONS

When (8) and (10) are combined, the expression for the concentration distribution becomes

$$\frac{C - C_W}{C_{avg} - C_W} = \frac{F(Sc, y^+)}{F(Sc, 20) + \sqrt{(2/f)} - 13{\cdot}0}. \quad (14)$$

Similarly the temperature distribution is obtained as

$$\frac{T - T_W}{T_{avg} - T_W} = \frac{F(Pr, y^+)}{F(Pr, 20) + \sqrt{(2/f)} - 13{\cdot}0}, \quad (15)$$

where

$$F(Pr, y^+) = \int_0^{y+} \frac{dy^+}{[1/Pr + \epsilon/\nu\,(y^+)]}. \quad (16)$$

Equations (14), (15) and (16) give the concentration and temperature distributions in the wall region of a pipe flow.

The functions $F(Sc$ or $Pr, y^+)$ and $F(Sc$ or Pr, 20) appearing in (13) and (15) involve the integrals which cannot be solved in closed forms. These were obtained by numerical integration using Simpson's one-third rule with a digital computer. These functions are shown in Table 1 for Sc or Pr numbers over a range of 0·1 to 10 000.

DISCUSSION

In Fig. 1 the function $F(Sc, y^+)$ or $F(Pr, y^+)$ is shown for the range of Schmidt and Prandtl

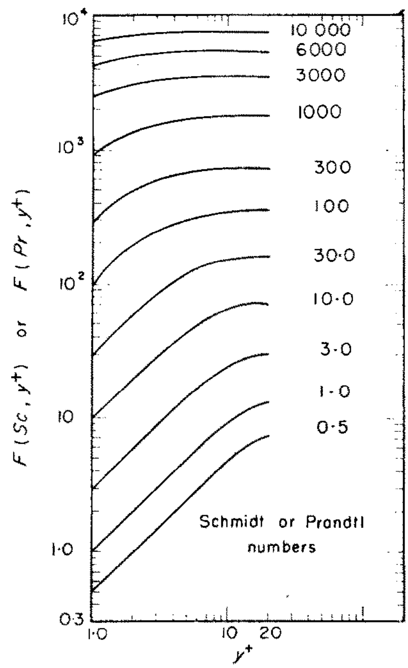

FIG. 1. Plot of $F(Sc, y^+)$ or $F(Pr, y^+)$ vs. y^+.

Table 1. Tabulation of functions $F(Sc, y^+)$ *or* $F(Pr, y^+) = \int_0^{y^+} \frac{dy^+}{\frac{1}{Sc} + \frac{\epsilon}{\nu}(y^+)}$

Sc or *Pr*	$y^+ \longrightarrow$ 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0·1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·5	1·6	1·7	1·7	1·8
0·2	0·2	0·4	0·6	0·8	1·0	1·2	1·4	1·6	1·8	2·0	2·1	2·3	2·5	2·7	2·8	3·0	3·1	3·2	3·3	3·3
0·3	0·3	0·6	0·9	1·2	1·5	1·8	2·1	2·4	2·6	2·9	3·2	3·4	3·7	3·9	4·1	4·3	4·5	4·6	4·7	4·8
0·4	0·4	0·8	1·2	1·6	2·0	2·4	2·8	3·1	3·5	3·9	4·2	4·5	4·8	5·1	5·4	5·6	5·8	5·9	6·0	6·1
0·5	0·5	1·0	1·5	2·0	2·5	3·0	3·4	3·9	4·4	4·8	5·2	5·6	6·0	6·3	6·6	6·8	7·0	7·2	7·3	7·4
0·6	0·6	1·2	1·8	2·4	3·0	3·6	4·1	4·7	5·2	5·7	6·2	6·6	7·0	7·4	7·7	8·0	8·2	8·4	8·5	8·6
0·7	0·7	1·4	2·1	2·8	3·5	4·1	4·8	5·4	6·0	6·6	7·2	7·7	8·1	8·5	8·8	9·1	9·3	9·5	9·6	9·7
0·8	0·8	1·6	2·4	3·2	4·0	4·7	5·5	6·2	6·9	7·5	8·1	8·6	9·1	9·6	9·9	10·2	10·4	10·6	10·8	10·8
0·9	0·9	1·8	2·7	3·6	4·5	5·3	6·1	6·9	7·7	8·4	9·0	9·6	10·1	10·6	11·0	11·3	11·5	11·7	11·8	11·9
1·0	1·0	2·0	3·0	4·0	4·9	5·9	6·8	7·7	8·5	9·3	10·0	10·6	11·1	11·6	12·0	12·3	12·6	12·8	12·9	13·0
1·1	1·1	2·2	3·3	4·4	5·4	6·5	7·5	8·4	9·3	10·1	10·9	11·5	12·1	12·6	13·0	13·3	13·6	13·8	13·9	14·0
1·2	1·2	2·4	3·6	4·8	5·9	7·0	8·1	9·1	10·1	11·0	11·8	12·5	13·1	13·6	14·0	14·3	14·6	14·8	14·9	15·0
1·3	1·3	2·6	3·9	5·2	6·4	7·6	8·8	9·9	10·9	11·8	12·6	13·4	14·0	14·5	15·0	15·3	15·6	15·8	15·9	16·0
1·4	1·4	2·8	4·2	5·6	6·9	8·2	9·4	10·6	11·6	12·6	13·5	14·3	14·9	15·5	15·9	16·3	16·5	16·7	16·9	17·0
1·5	1·5	3·0	4·5	5·9	7·4	8·8	10·1	11·3	12·4	13·4	14·4	15·2	15·8	16·4	16·9	17·2	17·5	17·7	17·8	17·9
1·6	1·6	3·2	4·8	6·3	7·9	9·3	10·7	12·0	13·2	14·3	15·2	16·0	16·7	17·3	17·8	18·1	18·4	18·6	18·7	18·8
1·7	1·7	3·4	5·1	6·7	8·3	9·9	11·3	12·7	13·9	15·1	16·0	16·9	17·6	18·2	18·7	19·0	19·3	19·5	19·7	19·8
1·8	1·8	3·6	5·4	7·1	8·8	10·4	12·0	13·4	14·7	15·9	16·9	17·7	18·5	19·1	19·6	19·9	20·2	20·4	20·6	20·7
1·9	1·9	3·8	5·7	7·5	9·3	11·0	12·6	14·1	15·4	16·6	17·7	18·6	19·3	19·9	20·4	20·8	21·1	21·3	21·4	21·5
2·0	2·0	4·0	6·0	7·9	9·8	11·6	13·2	14·8	16·2	17·4	18·5	19·4	20·2	20·8	21·3	21·7	22·0	22·2	22·3	22·4
2·1	2·1	4·2	6·3	8·3	10·3	12·1	13·9	15·5	16·9	18·2	19·3	20·2	21·0	21·6	22·1	22·5	22·8	23·0	23·2	23·3
2·2	2·2	4·4	6·6	8·7	10·7	12·7	14·5	16·2	17·6	19·0	20·1	21·0	21·8	22·5	23·0	23·4	23·7	23·9	24·0	24·1
2·3	2·3	4·6	6·9	9·1	11·2	13·2	15·1	16·8	18·4	19·7	20·9	21·8	22·7	23·3	23·8	24·2	24·5	24·7	24·8	24·9
2·4	2·4	4·8	7·2	9·5	11·7	13·8	15·7	17·5	19·1	20·5	21·6	22·6	23·5	14·1	24·6	25·0	25·3	25·5	25·7	25·8
2·5	2·5	5·0	7·5	9·9	12·2	14·3	16·4	18·2	19·8	21·2	22·4	23·4	24·3	24·9	25·4	25·8	26·1	26·3	26·5	26·6
2·6	2·6	5·2	7·7	10·2	12·6	14·9	17·0	18·8	20·5	21·9	23·2	24·2	25·0	25·7	26·2	26·6	26·9	27·1	27·3	27·4
2·7	2·7	5·4	8·0	10·6	13·1	15·4	17·6	19·5	21·2	22·7	23·9	25·0	25·8	26·5	27·0	27·4	27·7	27·9	28·1	28·2
2·8	2·8	5·6	8·3	11·0	13·6	16·0	18·2	20·2	21·9	23·4	24·7	25·7	26·6	27·3	27·8	28·2	28·5	28·7	28·9	29·0
2·9	2·9	5·8	8·6	11·4	14·0	16·5	18·8	20·8	22·6	24·1	25·4	26·5	27·4	28·1	28·6	29·0	29·3	29·5	29·6	29·8
3·0	3·0	6·0	8·9	11·8	14·5	17·1	19·4	21·5	23·3	24·8	26·2	27·2	28·1	28·8	29·4	29·8	30·1	30·3	30·4	30·5
3·1	3·1	6·2	9·2	12·2	15·0	17·6	20·0	22·1	24·0	25·6	26·9	28·0	28·9	29·6	30·1	30·5	30·8	31·0	31·2	31·3
3·2	3·2	6·4	9·5	12·6	15·5	18·1	20·6	22·8	24·6	26·3	27·6	28·7	29·6	30·3	30·9	31·3	31·6	31·8	31·9	32·0
3·3	3·3	6·6	9·8	12·9	15·9	18·7	21·2	23·4	25·3	27·0	28·3	29·4	30·4	31·1	31·6	32·0	32·3	32·5	32·7	32·8
3·4	3·4	6·8	10·1	13·3	16·4	19·2	21·8	24·0	26·0	27·7	29·0	30·2	31·1	31·8	32·3	32·8	33·1	33·3	33·4	33·5
3·5	3·5	7·0	10·4	13·7	16·9	19·8	22·4	24·7	26·7	28·3	29·7	30·9	31·8	32·5	33·1	33·5	33·8	34·0	34·2	34·3
3·6	3·6	7·2	10·7	14·1	17·3	20·3	23·0	25·3	27·3	29·0	30·4	31·6	32·5	33·2	33·8	34·2	34·5	34·7	34·9	35·0
3·7	3·7	7·4	11·0	14·5	17·8	20·8	23·5	25·9	28·0	29·7	31·1	23·3	33·2	34·0	34·5	34·9	35·2	35·5	35·6	35·7
3·8	3·8	7·6	11·3	14·9	18·2	21·3	74·1	26·6	28·6	30·4	31·8	33·0	33·9	34·7	35·2	35·7	36·0	36·2	36·3	36·4
3·9	3·9	7·8	11·6	15·3	18·7	21·9	24·7	27·2	29·3	31·1	32·5	33·7	34·6	35·4	35·9	36·4	36·7	36·9	37·0	37·1
4·0	4·0	8·0	11·9	15·6	19·2	22·4	25·3	27·8	29·9	31·7	33·2	34·4	35·3	36·1	36·6	37·1	37·4	37·6	37·7	37·8
4·1	4·1	8·2	12·2	16·0	19·6	22·9	25·9	28·4	30·6	32·4	33·9	35·1	36·0	36·8	37·3	37·8	38·1	38·3	38·4	38·5
4·2	4·2	8·4	12·5	16·4	20·1	23·4	26·4	29·0	31·2	33·1	34·6	35·8	36·7	37·5	38·0	38·5	38·8	39·0	39·1	39·2
4·3	4·3	8·6	12·8	16·8	20·5	24·0	27·0	29·6	31·9	33·7	35·2	36·4	37·4	38·2	38·7	39·1	39·5	39·7	39·8	39·9
4·4	4·4	8·8	13·1	17·2	21·0	24·5	27·6	30·2	32·5	34·4	35·9	37·1	38·1	38·8	39·4	39·8	40·1	40·4	40·5	40·6
4·5	4·5	9·0	13·3	17·5	21·5	25·0	28·1	30·8	33·1	35·0	36·5	37·8	38·8	39·5	40·1	40·5	40·8	41·0	41·2	41·3
4·6	4·6	9·2	13·6	17·9	21·9	25·5	28·7	31·4	33·8	35·7	37·2	38·4	39·4	40·2	40·8	41·2	41·5	41·7	41·9	42·0
4·7	4·7	9·4	13·9	18·3	22·4	26·0	29·3	32·0	34·4	36·3	37·9	39·1	40·1	40·8	41·4	41·9	42·2	42·4	42·5	42·6
4·8	4·8	9·6	14·2	18·7	22·8	26·6	29·8	32·6	35·0	36·9	38·5	39·8	40·7	41·5	42·1	42·5	42·8	43·0	43·2	43·3
4·9	4·9	9·8	14·5	19·1	23·3	27·1	30·4	33·2	35·6	37·6	39·2	40·4	41·4	42·2	42·7	43·2	43·5	43·7	43·9	44·0
5·0	5·0	10·0	14·8	19·4	23·7	27·6	31·0	33·8	36·2	38·2	39·8	41·1	42·1	42·8	43·4	43·8	44·1	44·4	44·5	44·6
5·1	5·1	10·2	15·1	19·8	24·2	28·1	31·5	34·4	36·8	38·8	40·4	41·7	42·7	43·5	44·1	44·5	44·8	45·0	45·2	45·3
5·2	5·2	10·4	15·4	20·2	24·6	28·6	32·1	35·0	37·4	39·5	41·4	42·3	43·3	44·1	44·7	45·1	45·4	45·7	45·8	45·9
5·3	5·3	10·6	15·7	20·6	25·1	29·1	32·6	35·6	38·1	40·1	41·7	43·0	44·0	44·8	45·3	45·8	46·1	46·3	46·5	46·6
5·4	5·4	10·8	16·0	20·9	25·5	29·6	33·2	36·2	38·7	40·7	42·3	43·6	44·6	45·4	46·0	46·4	46·7	46·9	47·1	47·2
5·5	5·5	11·0	16·3	21·3	26·0	30·1	33·7	36·7	39·3	41·3	42·9	44·2	45·3	46·0	46·6	47·1	47·4	47·6	47·7	47·8
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5·7	5·7	11·3	16·9	22·1	26·9	31·1	34·8	37·9	40·5	42·5	44·2	45·5	46·5	47·3	47·9	48·3	48·6	48·8	49·0	49·1
5·8	5·8	11·5	17·1	22·4	27·3	31·6	35·3	38·5	41·0	43·1	44·8	46·1	47·1	47·9	48·5	48·9	49·3	49·5	49·6	49·7
5·9	5·9	11·7	17·4	22·8	27·8	32·1	35·9	39·0	41·6	43·7	45·4	46·7	47·7	48·5	49·1	49·6	49·9	50·1	50·2	50·3

6·0	6·0	11·9	17·7	23·2	28·2	32·6	36·4	39·6	42·2	44·3	46·0	47·3	48·4	49·2	49·7	50·2	50·5	50·7	50·9	51·0
6·1	6·1	12·1	18·0	23·6	28·6	33·1	37·0	40·2	42·8	44·9	46·6	47·9	49·0	49·8	50·4	50·8	51·1	51·3	51·5	51·6
6·2	6·2	12·3	18·3	23·9	29·1	33·6	37·5	40·7	43·4	45·5	47·2	48·5	49·6	50·4	51·0	51·4	51·7	51·9	52·1	52·2
6·3	6·3	12·5	18·6	24·3	29·5	34·1	38·0	41·3	44·0	46·1	47·8	49·2	50·2	51·0	51·6	52·0	52·3	52·5	52·7	52·8
6·4	6·4	12·7	18·9	24·7	30·0	34·6	38·6	41·9	44·5	46·7	48·4	49·8	50·8	51·6	52·2	52·6	52·9	53·2	53·3	53·4
6·5	6·5	12·9	19·2	25·1	30·4	35·1	39·1	42·4	45·1	47·3	49·0	50·4	51·4	52·2	52·8	53·2	53·5	53·8	53·9	54·0
6·6	6·6	13·1	19·5	25·4	30·8	35·6	39·6	43·0	45·7	47·9	49·6	50·9	52·0	52·8	53·4	53·8	54·1	54·4	54·5	54·6
6·7	6·7	13·3	19·8	25·8	31·3	36·1	40·2	43·5	46·3	48·5	50·2	51·5	52·6	53·4	54·0	54·4	54·7	55·0	55·1	55·2
6·8	6·8	13·5	20·1	26·2	31·7	36·6	40·7	44·1	46·8	49·0	50·8	52·1	53·2	54·0	54·6	55·0	55·3	55·6	55·7	55·8
6·9	6·9	13·7	20·3	26·6	32·2	37·1	41·2	44·6	47·4	49·6	51·4	52·7	53·8	54·6	55·2	55·6	55·9	56·1	56·3	56·4
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7·1	7·1	14·1	20·9	27·3	33·0	38·0	42·2	45·7	48·5	50·7	52·5	53·9	54·9	55·7	56·3	56·8	57·1	57·3	57·5	57·6
7·2	7·2	14·3	21·2	27·7	33·5	38·5	42·8	46·3	49·1	51·3	53·1	54·5	55·5	56·3	56·9	57·4	57·7	57·9	58·1	58·2
7·3	7·3	14·5	21·5	28·0	33·9	39·0	43·3	46·8	49·6	51·9	53·7	55·0	56·1	56·9	57·5	57·9	58·3	58·5	58·6	58·7
7·4	7·4	14·7	21·8	28·4	34·3	39·5	43·8	47·3	50·2	52·4	54·2	55·6	56·7	57·5	58·1	58·5	59·8	59·1	59·2	59·3
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7·7	7·7	15·3	22·7	29·5	35·6	40·9	45·3	49·0	51·8	54·1	55·9	57·3	58·4	59·2	59·8	60·2	60·6	60·8	60·9	61·0
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9·2	9·2	18·3	27·0	35·0	42·0	48·0	52·9	56·8	59·9	62·3	64·1	65·6	66·7	67·5	68·1	68·5	68·9	69·1	69·2	69·3
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9·9	9·9	19·6	29·0	37·5	45·0	51·2	56·3	60·3	63·5	65·9	67·8	69·3	70·4	71·2	71·8	72·3	72·6	72·8	72·9	73·1
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40	39	77	109	134	151	163	171	177	181	184	186	188	189	190	191	191	192	192	192	192
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70	69	133	182	215	236	250	259	265	270	273	275	277	278	279	280	280	280	281	281	281
80	79	151	205	240	262	276	286	292	296	299	302	303	305	305	306	307	307	307	307	307
90	89	168	227	264	287	301	311	317	322	325	327	329	330	331	331	332	332	332	333	333
100	99	186	249	288	311	326	335	341	346	349	351	353	354	355	356	356	357	357	357	357
200	196	352	444	493	520	535	545	552	556	560	562	564	565	566	566	567	567	567	568	568
300	291	504	614	667	695	711	721	728	732	735	738	739	741	742	742	743	743	743	743	744
400	385	645	767	822	851	867	877	884	889	892	894	896	897	898	899	899	900	900	900	900
500	477	778	908	965	994	1011	1021	1028	1033	1036	1038	1040	1041	1042	1043	1043	1043	1044	1044	1044
600	568	905	1041	1099	1128	1145	1155	1162	1167	1170	1172	1174	1175	1176	1177	1177	1178	1178	1178	1178
700	657	1026	1166	1226	1255	1272	1282	1289	1294	1297	1299	1301	1302	1303	1304	1304	1305	1305	1305	1305
800	746	1142	1286	1346	1376	1393	1403	1410	1415	1418	1420	1422	1423	1424	1425	1425	1426	1426	1426	1426
900	833	1254	1402	1462	1492	1509	1519	1526	1531	1534	1536	1538	1539	1540	1541	1541	1542	1542	1542	1542
1000	919	1363	1513	1574	1604	1621	1631	1638	1643	1646	1648	1650	1651	1652	1653	1653	1654	1654	1654	1654
2000	1726	2318	2482	2545	2575	2592	2603	2610	2614	2618	2620	2622	2623	2624	2624	2625	2625	2625	2626	2626
3000	2457	3129	3299	3362	3393	3410	3420	3427	3432	3435	3438	3439	3440	3441	3442	3443	3443	3443	3443	3443
4000	3131	3854	4026	4090	4121	4138	4149	4155	4160	4163	4166	4167	4169	4169	4170	4171	4171	4171	4171	4171
5000	3759	4517	4690	4755	4785	4803	4813	4820	4824	4828	4830	4832	4833	4834	4835	4835	4835	4836	4836	4836
6000	4346	5130	5305	5370	5400	5418	5428	5435	5440	5443	5445	5447	5448	5449	5450	5450	5450	5451	5451	5451
7000	4900	5704	5880	5944	5975	5992	6003	6009	6014	6017	6020	6021	6023	6024	6024	6025	6025	6025	6025	6026
8000	5424	6244	6420	6485	6516	6533	6543	6550	6555	6558	6560	6562	6563	6564	6565	6565	6566	6566	6566	6566
9000	5921	6754	6931	6996	7027	7044	7054	7061	7066	7069	7071	7073	7074	7075	7076	7076	7077	7077	7077	7077
10000	6395	7240	7417	7481	7512	7529	7540	7547	7551	7555	7557	7559	7560	7561	7561	7562	7562	7563	7563	7563

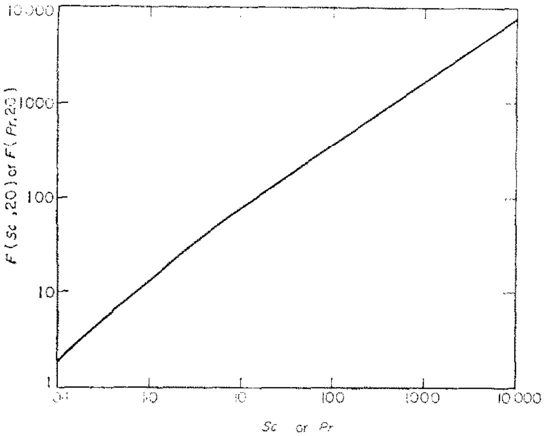

FIG. 2. Plot of $F(Sc)$ vs. Sc or $F(Pr)$ vs. Pr.

numbers from 0·5 to 10 000. Fig. 2 shows the calculated values of the function $F(Sc, 20)$ or $F(Pr, 20)$. For rapid calculation purposes the function shown on Fig. 2 can be approximated with a maximum error of ± 2 per cent as follows:

For turbulent diffusion in gases the function $F(Sc, 20)$ can be represented by the equation

$$F(Sc, 20) = 13{\cdot}0\,(Sc)^{0{\cdot}80}, \text{ where } 0{\cdot}2 \leqslant Sc \leqslant 2. \tag{17}$$

For diffusion in liquid streams the following equation is proposed:

$$F(Sc, 20) = 17{\cdot}25\,(Sc)^{0{\cdot}66}, \text{ where } 100 \leqslant Sc \leqslant 10\,000. \tag{18}$$

For the intermediate range of Schmidt and Prandtl numbers the function F can be given by the equation

$$F(Sc, 20) = 13{\cdot}8\,(Sc)^{0{\cdot}71}, \text{ where } 2 \leqslant Sc \leqslant 100. \tag{19}$$

In Figs. 3 and 4 the proposed mass- and heat-transfer Stanton-number relationship is compared with heat- and mass-transfer data of many workers at Reynolds numbers of 10 000 and 25 000. The experimental points representing a mean through the data are the ones chosen by Deissler except for the datum point of Meyerink and Friedlander [11] which was taken from their mean curve through the data. The proposed expression agrees very well with the data over a wide range of Schmidt and Prandtl numbers.

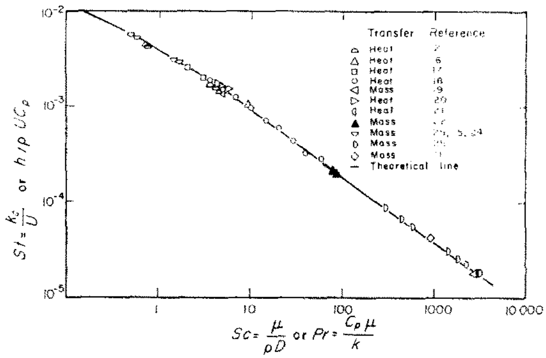

FIG. 3. Comparison of present theory with experimental data at a Reynolds number of 10 000.

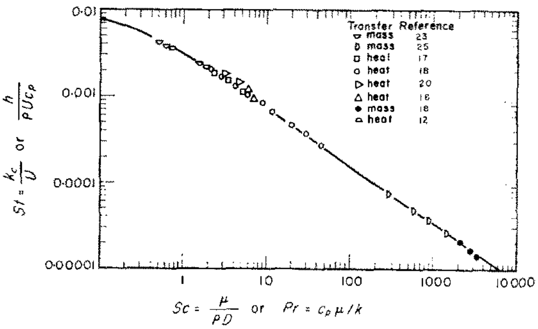

FIG. 4. Comparison of present theory with experimental data at a Reynolds number of 25 000.

Fig. 5 shows a comparison of the values calculated from the proposed Stanton number relationship at a Reynolds number of 50 000 with those calculated from the expressions of Deissler [10], Gowariker [6], Lin, Moulton and Putnam [13], Rannie [7], and von Kármán [14]. It is noted that von Kármán's relationship

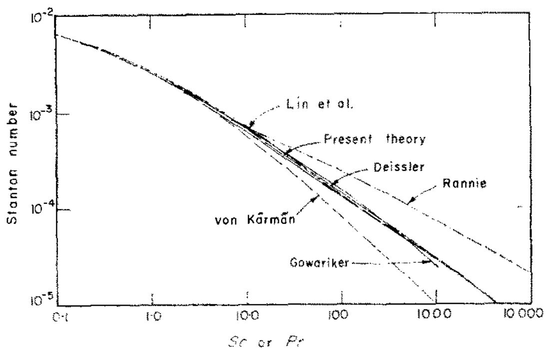

FIG. 5. Comparison of several analogies at a Reynolds number of 50 000.

gives too low results and Rannie's expression gives too high results at large Schmidt and Prandtl numbers. However, our correlation agrees very well with those of Lin, Moulton and Putnam, and Deissler.

By combinations of (12) and (18), we express Stanton number in terms of Schmidt number and friction factor for large values of Schmidt number by

$$St_m = 0{\cdot}058\ \sqrt{(f/2)}\ (Sc)^{-0{\cdot}66}. \qquad (20)$$

It is of interest that the exponent on the Schmidt number as given by (20) is in agreement with the Chilton and Colburn analogy [15].

The curves for heat and mass transfer expressed in the form of Nusselt numbers are shown in Fig. 6. The Nusselt number for mass transfer (i.e. the Sherwood number) can be correlated by the following expressions:

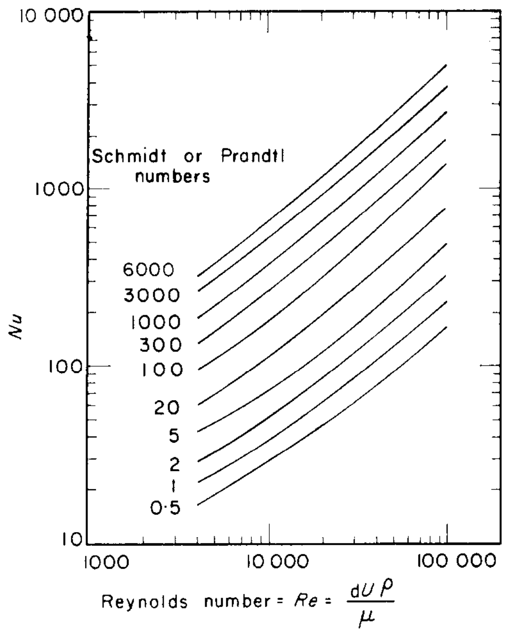

FIG. 6. Nusselt number vs. Reynolds number at various Schmidt or Prandtl numbers.

$$Nu = \frac{(f/2)\,(Re)\,(Sc)}{1 + \sqrt{(f/2)}\,[13{\cdot}0\,(Sc)^{0{\cdot}80} - 13{\cdot}0]}$$

for $0{\cdot}2 \leqslant Sc \leqslant 2$, (21)

$$Nu = \frac{(f/2)\,(Re)\,(Sc)}{1 + \sqrt{(f/2)}\,[13{\cdot}8\,(Sc)^{0{\cdot}71} - 13{\cdot}0]}$$

for $2 \leqslant Sc \leqslant 100$, (22)

and $Nu = (0{\cdot}058)\ \sqrt{(f/2)}\ (Re)\ (Sc)^{0{\cdot}34}$

for $100 \leqslant Sc \leqslant 10\,000$. (23)

Heat transfer Nusselt numbers can be obtained by replacing Schmidt numbers by Prandtl numbers in the above expressions. These expressions for Nusselt numbers are based on the difference between wall and average concentration or temperature.

CONCLUSIONS

We show that agreement between predicted values and experimental data of mass- and heat-transfer rates supports the use of the proposed eddy-viscosity distribution function. Our predicted transfer rates agree very well with those calculated from Lin, Moulton and Putnam relationships, which are in excellent agreement with experimental data. Our proposed eddy-viscosity expression is useful because it applies to the whole wall region of pipe flow and is developed on a sound theoretical basis. It would also appear that the previous concept of sharply defined fluid layers is not necessary.

To calculate mass- and heat-transfer rates, simplified equations in the form of equations (17), (18) and (19) can be used to predict the complicated functions $F(Sc)$ and $F(Pr)$. Also, simplified equations in the form of equations (21), (22) and (23) are proposed to predict heat- and mass-transfer Nusselt-number relationships.

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Résumé—Les auteurs présentent une nouvelle expression du transport de chaleur et de masse pour un fluide en écoulement pleinement turbulent dans une conduite. Cette expression diffère des relations empiriques précédentes en ce qu'elle est établie à partir d'une distribution de viscosité turbulente continue de la paroi vers le centre de la conduite. Les coefficients de transport calculés à partir de la nouvelle corrélation sont en excellent accord avec les données expérimentales de transport de chaleur et de masse dans les écoulements fluides.

Zusammenfassung—Zum Beschreiben des Stoff- und Wärmeüberganges an ein Fluid in voll entwickelter turbulenter Rohrströmung wird eine neue Beziehung angegeben. Von älteren empirischen Beziehungen unterscheidet sie sich dadurch, dass sie auf einer theoretischen kontinuirlichen Verteilung der Wirbelviskosität von der Rohrwand bis zur Achse beruht. Mittels dieser neuen Beziehung berechnete Übergangswerte stimmen mit Versuchsergebnissen ausgezeichnet überein.

Аннотация—Дается новое соотношение для описания процесса переноса тепла и вещества к жидкости при полностью развитом турбулентном течении в трубе. Это соотношение отличаестя от предыдущих эмпирических соотношений тем, что оно основано на теоретическом распределении турбулентной вязкости от стенки к центру трубы. Скорости переноса, вычисленные с помощью новой корреляции, хорошо согласуются с экспериментальными данными по переносу тепла и массы к потокам жидкости.